Stochastic

9/11/2015 الاثنين م. مجال 3 سكشن

(10) the probability that a man lives 10 years $\frac{1}{4}$ (A)

the " " his wife " " " $\frac{1}{3}$ (B)

find probability that

i) both be alive

$$P(A \cap B) = P(A)\,P(B)$$
$$= \frac{1}{3} * \frac{1}{4} = \frac{1}{12}$$

ii) at least one will live

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12}$$

iii) Neither will be alive

$$P(A \cup B)^c = 1 - P(A \cup B)$$

vi) only the wife

$$P(B - A) = P(B) - P(A \cap B)$$

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(11) A ⟶ family has children of both gender

B ⟶ at most one boy

① Show that A & B are independent ⟶ 3 children

$S = \{(bbb), (bbg), (bgb), (bgg), (gbb), (gbg), (ggb), (ggg)\}$

$$P(A \cap B) = P(A)\,P(B)$$

$P(A) = \frac{6}{8}$, $P(B) = \frac{4}{8}$, $P(A \cup B) = \frac{3}{8}$

$$P(A)\,P(B) = \frac{24}{64} = \frac{3}{8}$$

② for 2 children, prove dependency

$S = \{(bb), (bg), (gb), (gg)\}$

$P(A) = \frac{2}{4}$, $P(B) = \frac{3}{4}$, $P(A \cap B) = \frac{2}{4}$

$P(A \cap B) \neq P(A)\,P(B)$

$\Rightarrow$ dependent

Report 3, 8, 12, 13, 17 Sheet 3

Stochastic Variables

* Density function: ($P(x)$ or $f(x)$) → continuous, discrete

1) $0 \leq f(x) \leq 1$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ → for $x \Rightarrow$ continuous random Var.

$\sum_{-\infty}^{\infty} f(x) = 1$ → for $x \Rightarrow$ discrete random Var.

* Commulative distribution function:-

$F(x) = P(X \leq x)$ الرقم اللي أصغر منه

at most 3

$P(X \leq 3) = P(3) + P(2) + P(1)$

$F(x)$ has to be increasing fn

① $F(\infty) = 1$ ② $F(-\infty) = 0$

③ $F(a) < F(b)$ if $a < b$

$$f(x) = \frac{dF(x)}{dx} \; ; \quad F(x) = \int_{-\infty}^{x} f(x)\,dx$$

... >> Sheet _4 << ...

① Let $f(x) = \begin{cases} x & 0 \le x \le 1 \\ \frac{3-x}{4} & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$

- prove that $f(x) \longrightarrow$ density fn
- Find $F(x)$

* $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow$ density fn

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{3} \frac{3-x}{4}\,dx + \int_{3}^{\infty} 0\,dx$$

$$= \frac{x}{2}\Big|_0^1 + \left(\frac{3x}{4} - \frac{x}{8}\right)\Big|_1^3 = 1$$

* for $-\infty \le x \le 0$

$$F(x) = \int_{-\infty}^{x} 0\,dx + F(-\infty) = 0$$

= for $0 \le x \le 1$

$$F(x) = F(0) + \int_{0}^{x} f(x)\,dx = 0 + \int_{0}^{x} x\,dx = \frac{x^2}{2}\Big|_0^x$$

$$= \frac{x^2}{2}$$

- for $1 \le x \le 3$

$$F(x) = F(1) + \int_{1}^{x} \left(\frac{3}{4} - x\right) dx = \frac{1}{2} + \left(\frac{3}{4}x - \frac{x^2}{2}\right)\Big|_1^x$$

$$= \frac{1}{2} + \frac{3}{4}x - \frac{x^2}{8} - \left(\frac{3}{4} - \frac{1}{2}\right) = \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8}$$

- for $3 \leq x \leq \infty$

$$F(x) = F(3) + \int_3^x 0 \, dx = 1$$

$$\therefore F(x) = \begin{cases} 0 & -\infty \leq x < 0 \\ x^2/2 & 0 \leq x \leq 1 \\ 3\frac{1}{4} - x^2/8 - 1/2 & 1 \leq x \leq 3 \\ 1 & 3 \leq x \end{cases}$$

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(2) If $f(x) = \frac{1}{2^x}$ for $x = 1, 2, 3, \ldots$

Can this $f_n \longrightarrow$ probability f_n

$$\sum_{-\infty}^{\infty} f(x) = 1/2 + 1/4 + 1/8 + 1/16 + \ldots \neq 1$$

not probability f_n

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(4) if $f(x) = \begin{cases} K(2-x) & 0 \leq x \leq 2 \\ 0 & \text{other wise} \end{cases}$

$\longrightarrow$ probability f_n , find K

$$\int_{-\infty}^{\infty} f(x) \, dx = 1$$

$$\int_0^2 (2K - xK) \, dx = 2Kx - \frac{x^2}{2}K \Big|_0^2 = 1$$

$$4K - 2K = 1$$

$$K = 1/2$$

مسألة 5 نفس الفكرة